Nonleptonic Hyperon Decays with QCD Sum Rules

E.M. Henley
Department of Physics and Institute for Nuclear Theory,
Box 351560, University of Washington, Seattle, Washington 98195

W-Y.P. Hwang

Department of Physics, National Taiwan University,
Taipei 106, Taiwan

L.S. Kisslinger

Department of Physics, Carnegie-Mellon University,
Pittsburgh, Pennsylvania 15213

Abstract

Despite measurements which date more than 20 years ago, no straightforward solution of the ratio of the parity-conserving (P-wave) to parity-violating (S-wave) decays of the hyperons has been obtained. Here we use two 2-point methods in QCD sum rules to examine the problem. We find that resonance contributions are needed to fit the data, similar to a chiral perturbation theory treatment.

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1 Introduction

The nonleptonic decays of the hyperons occur with pion emission, e.g., $\Lambda^0 \to p + \pi^-$. Measurements of the decay rates and the S/P (pv/pc) ratios of the emitted pions were carried out over 20 years ago [1]. They remain of interest today because no one has been able to provide a relatively simple explanation of the S/P ratios.

To-date a variety of approaches have been used. Some of the early work used a soft pion approach [1]. In this limit the $\Sigma^+ \to n\pi^+$ decay with an S-wave pion vanishes. The $\Sigma^- \to n\pi^-$ decay amplitude can be obtained approximately by an adjustment of the SU(3) F/D ratio. The soft pion approach in the S-wave and poles in the P-waves approach (see Fig.1) was used by Donoghue et al.[1], who argue that there could also be a direct coupling, as shown in Fig. 1d, but they too have difficulty in fitting the S/P ratios. Other work is that of ref. [2]. Most recently, Barasoy and Holstein [3] have used chiral perturbation theory, but have had to include $(70, 1^-)\frac{1}{2}$ resonances and parameters to obtain a reasonable fit to the data.

Of the seven decays, $\Sigma^+ \to p\pi^0$, $\Sigma^+ \to n\pi^+$, $\Sigma^- \to n\pi^-$, $\Lambda^0 \to n\pi^0$, $\Lambda^0 \to p\pi^-$, $\Xi^- \to \Lambda^0\pi^-$, $\Xi^0 \to \Lambda^0\pi^0$, there are only four independent ones if isospin symmetry holds. Experimentally, the SU(3) 27-plet is smaller than the octet by a factor of approximately 20. Like those before us, we choose the 4 independent decays as those with a charged pion, namely

$$\begin{aligned}
\Sigma^+_+ &: \quad \Sigma^+ \to n\pi^+, \\
\Sigma^-_- &: \quad \Sigma^- \to n\pi^-, \\
\Lambda^0_- &: \quad \Lambda^0 \to p\pi^-, \\
\Xi^-_- &: \quad \Xi^- \to \Lambda^0\pi^-.
\end{aligned} \tag{1}$$

In the present work we use the method of QCD sum rules with two 2-point formulations for the three-point correlators needed to obtain coupling constants, which we discuss in the next section. We find that in order to find stable solutions for the sum rules we must explicitly introduce single-pole resonance contributons, analogous to the addition of resonance contributions in Ref. [3]. Since this introduces new constants which can, however, be used to fit the data, we did not proceed to investigate the last two decays (Λ^0_- and Ξ^-_-).

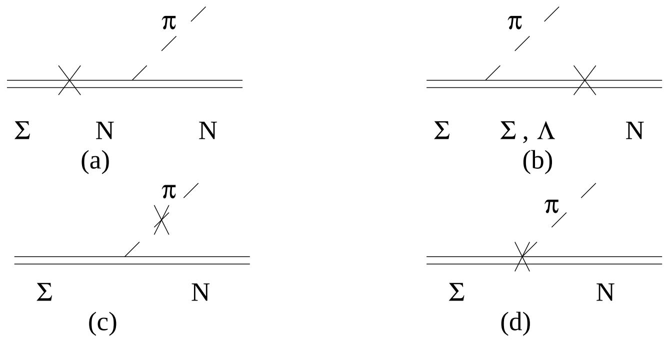

Fig. 1 Sigma decay into a nucleon and a pion via soft pion approach.

2 Methodology

2.1 QCD Sum Rules

QCD sum rules were introduced by Shifman, Vainshtein, and Zhakarov [4]. It is a useful method to obtain properties of hadrons and it uses QCD explicitly. The short range perturbative QCD is extended by an operator product expansion (OPE) of the correlator, giving a series in inverse powers of the squared momentum with Wilson coefficients. The convergence at low momentum is improved by using a type of Laplace transform, called the Borel transform. The coefficients involve universal quark and gluon condensates. This quark–based calculation of a given correlator is equated to the same correlator obtained via a dispersion relation, giving sum rules from which a property can be estimated. The method can be extended for quantities in an external field, such as the magnetic coupling to a nucleon in an electromagnetic field [5].

The method begins with a correlator

$$\Pi(p) = i \int d^4x \; e^{iq\cdot x} < 0 \mid T[\eta(x)\bar{\eta}(0)] \mid 0 > , \tag{2}$$

where η has the quantum numbers of the hadron being studied. For a proton, we may take

$$\begin{aligned} \eta(x) &= \epsilon^{abc}[u^{aT}(x)C\gamma_\mu u^b(x)]\gamma^5\gamma^\mu d^c(x) , \\ \bar{\eta} &= \epsilon^{abc}[\bar{u}^b\gamma_\nu C\bar{u}^{aT}]\bar{d}^c\gamma^\nu\gamma^5 , \end{aligned} \tag{3}$$

where a, b, c are color indices and the notation of Bjorken and Drell is used. The quark field operators d, u, s destroy these quarks, C stands for charge conjugation and T for transpose.

The "currents" η are not unique[6], but the form given in Eq. (3) has been used by many authors. The correlator can be written as an operator product expansion

$$\Pi = C_s I + \sum_n C_n(p^2) O_n \tag{4}$$

where the operators O_n can be ordered by dimension and the corresponding Wilson coefficients decrease by increasing powers of p^2.

The correlators Π have structure functions Π^j, each of which satisfies a dispersion relation ($P^2 = -p^2$)

$$\Pi^j(P^2) = \frac{1}{\pi} \int_0^\infty \frac{Im\Pi^j(s) ds}{s + P^2} \tag{5}$$

Subtraction terms in Eq.(4) are eliminated by means of a Borel transform, which guarantees convergence,

$$\begin{aligned} B[F(p^2)] &= \lim_{\substack{n\to\infty \\ -p^2\to\infty\,,\, -p^2/n\to M_B^2}} (-p^2)^{(n+1)} (\frac{d}{d(-p^2)})^n F(p^2) \\ &= \frac{1}{\pi} \int_0^\infty ds\ ImF(s) e^{-s/M_B^2}\ . \end{aligned} \tag{6}$$

There are a number of ways to use these sum rules:

(i) a two-point method with or without an external field;

(ii) a three-point method with couplings and momentum transfers considered explicitly. This method has fewer susceptibilities but it is more complicated; it may require non–local condensates.

In this article we will use only two-point methods. We will compare the two point method in an external field with the two point method with a pion creation matrix element.

2.2 Σ^-_- in an External Pion Field

The calculation of the non-leptonic decays of hyperons is similar to that of the weak pion-nucleon coupling constant. If we neglect the mass difference in the baryon octet, then $\Sigma \rightarrow N\pi$ is quite akin to $N \stackrel{weak}{\rightarrow} N\pi$. The primary difference is that the latter is due to weak neutral currents and the former due to charged currents.

We use the operators

$$\begin{aligned} \eta_{\Sigma^-} &= \epsilon^{abc}[d^{aT}C\gamma_\mu d^b]\gamma^5\gamma^\mu s^c \ , \\ \eta_n &= \epsilon^{abc}[\bar{d}^b\gamma_\nu C\bar{d}^{aT}]u^c\gamma^\nu\gamma^5 \end{aligned} \tag{7}$$

for the Σ^- and for the n. In addition, we need the weak interaction, for which we use the local one,

$$\begin{aligned} H_W &= \frac{G_F}{\sqrt{2}}J^\mu J^\dagger_\mu \ , \\ J^\mu &= \bar{u}\gamma^\mu(1-\gamma_5)s \ sin\theta_C + \bar{u}\gamma^\mu(1-\gamma_5)d \ cos\theta_C \ , \end{aligned} \tag{8}$$

where G_F is the Fermi coupling constant and θ_C is the Cabibbo angle. The correlator is

$$\Pi = i\int d^4x\ e^{ip\cdot x} < 0 \mid T[\eta_\Sigma(x)\, H_W\, \bar{\eta}_n(x)] \mid 0 >_\pi, \tag{9}$$

where the quarks propagate in an external field. Since the point at which the external pion field is at zero momentum transfer[5], the three-point function for the vertex is reduced to a two-point function. To the order considered here, the QCD diagrams which contribute to the correlator are shown in Fig.2. In the diagrams, the wavy line represents a $W^\pm$ boson, the dashed line represents a pion. The diagrams are evaluated in momentum space. Fig. (2a) gives no contribution. Diagram (2e) is quite different from the others. It involves the weak matrix element

$$< \pi^- \mid J_\alpha \mid \pi^0 >= \sqrt{2}F_\pi q_\alpha \ , \tag{10}$$

where F_π is the weak pion form factor and q is the momentum of the π^0. The contribution of this diagram cannot be neglected. There are additional

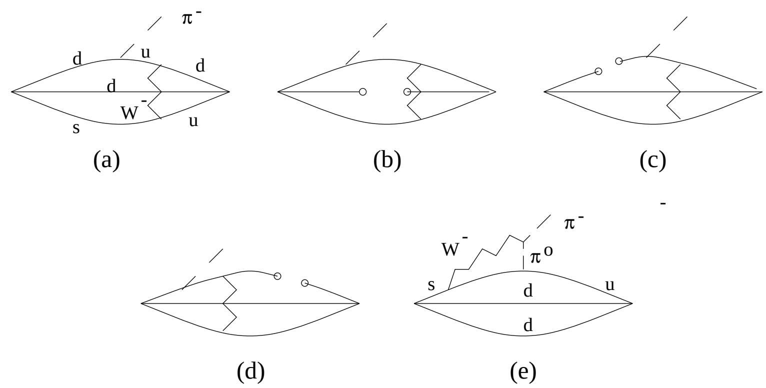

Fig. 2 Processes for Σ^- decay into a neutron and an external π^- field

higher order terms, e.g., gluonic corrections, which we omit here. Reasonjs will become apparent further on.

We obtain the following results, using dimensional regularization in $4-\epsilon$ dimensions

$$
\begin{aligned}
Fig.2b &: \; -\frac{16A < \bar{q}q > p^2 \ln p^2}{(4\pi)^4\epsilon}[\not{p}(1-\gamma\epsilon+\frac{43}{24}\epsilon-\frac{\epsilon}{2}\ln p^2)+4m(1-\gamma\epsilon+\frac{15}{8}\epsilon-\frac{\epsilon}{2}\ln P^2)] \\
&(1-\gamma_5) \; , \\
Fig.2c &: \; \frac{4A < \bar{q}q > p^2 \ln p^2}{(4\pi)^4\epsilon}[\not{p}(1-\gamma\epsilon+\frac{8}{3}\epsilon-\frac{\epsilon}{2}\ln p^2)+4m(1-\gamma\epsilon+\frac{11}{4}\epsilon-\frac{\epsilon}{2}\ln p^2)] \\
&(1-\gamma_5) \\
Fig.2d &: \; \frac{4A < \bar{q}q > p^2 \ln p^2}{(4\pi)^4}[\frac{5}{9}\not{p}-\frac{13}{4\epsilon}m(1-\gamma\epsilon+\frac{355}{156}\epsilon-\frac{13\epsilon}{8}\ln p^2)](1-\gamma_5) \; , \qquad (11) \\
Fig.2e &: \; \frac{\sqrt{2}F_\pi p^4 A \ln p^2}{(4\pi)^6 6}[p^2+\frac{6m\not{p}}{\epsilon}(1+\frac{7}{2}\gamma\epsilon+\frac{\epsilon}{6}-\frac{9}{8}\epsilon\ln p^2)](1+\gamma_5) \; ,
\end{aligned}
$$

where γ is the Euler constant, m the strange quark mass, and

$$
A = \sqrt{2}G_F \sin\theta_C \cos\theta_C \; . \qquad (12)
$$

From these equations, it is clear that, as in determining the weak pion nucleon coupling constant [7], we need to include vertex renormalizations. There are several of these, shown in Fig. 3. For Fig. 3a we obtain

$$
\begin{aligned}
\bar{\eta} &= \bar{u}\Gamma_\nu C \bar{s}\gamma^\nu\gamma_5 \; , \qquad (13) \\
\Gamma_\nu(3a) &= \frac{A}{(4\pi)^2\epsilon}(1-\frac{\gamma\epsilon}{2}+\frac{\epsilon}{2})\gamma_\nu(k^2)^{1-\epsilon/2}(1-\gamma_5) \; .
\end{aligned}
$$

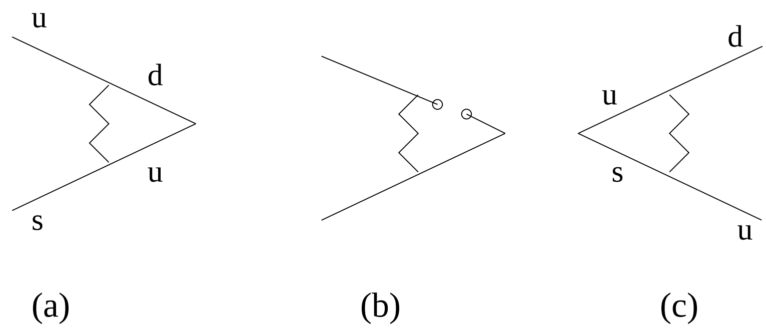

Fig. 3 Vertex renormalization diagrams

For Fig 3b we get

$$\Gamma_\nu(3b) = -\frac{A}{6}(1-\frac{\epsilon}{2}) < \bar{q}q > \frac{\not k}{k^2}\gamma_\nu(1-\gamma_5) \; . \tag{14}$$

For Fig. 3c we obtain

$$\begin{aligned} \eta &= d^T C\gamma^\nu \Gamma_\nu u \; , \\ \Gamma_\nu(c) &= -\frac{8Am}{(4\pi)^2\epsilon}\gamma_\nu(1-\frac{\gamma\epsilon}{2}+\frac{\epsilon}{2})(k^2)^{-\epsilon/2}(1-\gamma_5) \; . \end{aligned} \tag{15}$$

Employing these 3 vertex renormalizations and calculating the diagrams of Fig.2 with them, we obtain

$$\Pi = -\frac{4Ap^2\ln p^2}{(4\pi)^4}\{[\frac{35}{18}\not p + \frac{19}{6}m] < \bar{q}q > (1-\gamma_5) - [\sqrt{2}\frac{F_\pi p^4}{24(4\pi)^2}+\ldots](1+\gamma_5)\} \; . \tag{16}$$

We have not carried out the renormalization for Fig. 2c because we shall see that it is not needed.

For the phenomenological (or so-called right-hand) side we have

$$\begin{aligned} \Pi = &-\lambda_N\lambda_\Sigma\frac{1}{\not p - M_N}(A_S+A_P\gamma_5)\frac{1}{\not p - M_\Sigma} \\ &+\frac{1}{\not p - M_N}(\bar{c}_1+\bar{c}_2\gamma_5)\frac{1}{\not p - M^*} \; , \end{aligned} \tag{17}$$

plus the continuum. The inclusion of resonances is indicated by the $(\bar{c}_1+\bar{c}_2\gamma_5)$ term; the mass M^* represents the resonance mass. The resonance terms can

be separated into two parts, each of which is a single pole term; we use $\frac{\bar{c}_a^- + \bar{c}_b^- \gamma_5}{M(\not{p} - \bar{M})}$ for these single pole terms. The choice of $\bar{M}$ is arbitrary; any other choice will simply lead to different values for $\bar{c}_a^-$ and $\bar{c}_b^-$. The single-pole quantities $\bar{c}_a^-, \bar{c}_b^-$ are functions of momentum, or after the Borel transform they are called c_a^-, c_b^- and are functions of the Borel mass, M_B. We define $\bar{M} \equiv \frac{1}{2}(M_N + M_\Sigma)$ and $\triangle M \equiv M_\Sigma - M_N$. To first order in $\triangle M$ we find

$$\Pi = -\frac{\lambda_N \lambda_\Sigma}{(p^2 - \bar{M}^2)^2}[A_S(p^2 + \bar{M}^2) - A_P\gamma_5(p^2 - \bar{M}^2) + A_S 2\bar{M} \not{p} + A_P \triangle M \not{p}\gamma_5] + \lambda_N \lambda_\Sigma \frac{(\not{p} + \bar{M})(c_a^- + c_b^- \gamma_5)}{M(p^2 - \bar{M}^2)} . \quad (18)$$

We abandon the $\not{p}$ and $\not{p}\gamma_5$ sum rules because $A_P \triangle M$ would vanish in the analogous pion-nucleon vertex sum rule; this term is too sensitive to $\triangle M$. The single pole term within the square bracket is similar to that which occurs in determining the pion-nucleon coupling constant. Fig. 2e requires no renormalization for the p^2 term; we have not carried out the renormalization for the $m \not{p}$ and $m \not{p}\gamma_5$ terms. After inclusion of the continuum and carrying out a Borel transform we obtain

$$\begin{aligned}\Pi &= -\frac{A}{4^5\pi^6 L^{4/9}}[\frac{76}{6}M_B^4 am E_2(1-\gamma_5) + \sqrt{2}\frac{M_B^8 E_3}{4}F_\pi(1+\gamma_5)] \\ &= -\lambda_N\lambda_\Sigma[A_S(\frac{2\bar{M}^2}{M_B^2} - 1) + A_P\gamma_5 + (c_a^- + c_b^-\gamma_5)]e^{-\frac{\bar{M}^2}{M_B^2}} , \end{aligned} \quad (19)$$

where E_n represents the continuum contribution,

$$E_n = 1 - (1 + x + \frac{x^2}{2} + \ldots + \frac{x^n}{n!})e^{-x} , \quad (20)$$

with $x = s/M_B^2$, where s is the continuum threshold. For the nucleon it was found that $s \approx 2.3 GeV^2$ and for the $\Sigma' s \approx 3.2 GeV^2$ [8]; here we usually take an intermediate value of $s = 2.8 GeV^2$ for the transition, but explore other values for stability. λ_N and λ_Σ are known from previous studies [8]: $\tilde{\lambda}_N \tilde{\lambda}_\Sigma = (2\pi)^4 \lambda_n \lambda_\Sigma = 0.303 GeV^6$

A crucial point in the analysis of the sum rules to to recognize that there is some uncertainty in the p-dependence of the single-pole terms. The assumption used in Eq.(19) is the simplest possible. There almost certainly

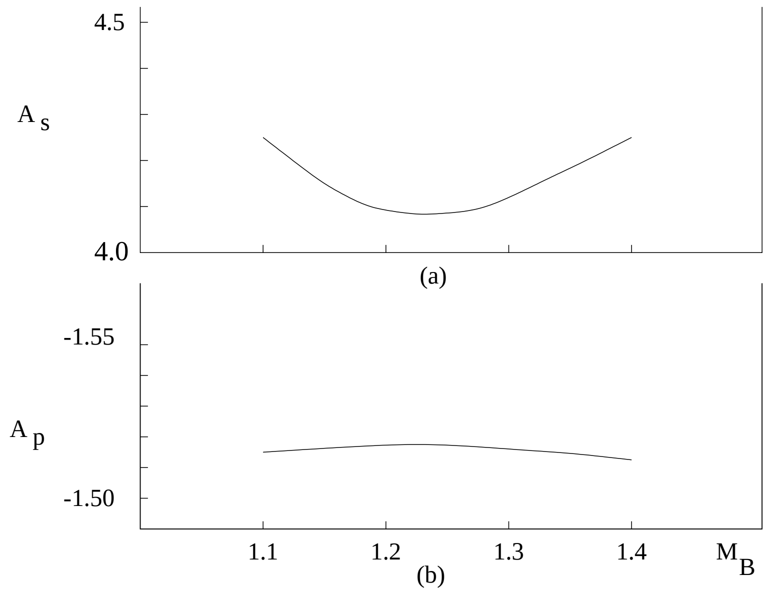

Fig. 4 Σ^-_- decay (a) A_s , (b) A_p

is other M_B dependence of the pole term, which makes no difference at the point $M_B = \bar{M}$, but which can give stable solutions. We find the reasonable but not obvious behavior that in every case the single-pole constants are linear functons of M_B.

As for the pion-nucleon weak coupling constant, we gain some stability by multiplying both sides of the QCD sum rules by M_B^2 and carrying out $(1 - M_B^2 \partial/\partial M_B^2) M_B^2\ \Pi$. This removes the great sensitivity in A_S to M_B from the double pole on the right-hand side, although some remains. We obtain

$$A_S = -\frac{\frac{A}{4^3\pi^2 L^{4/9}}(\frac{76}{3}M_B^6 amE_2 + \sqrt{2}E_4 M_B^{10} F_\pi) - \tilde{\lambda}_N \tilde{\lambda}_\Sigma c_a^- \bar{M}^2 e^{-M^2/M_B^2}}{\tilde{\lambda}_N \tilde{\lambda}_\Sigma [2(1 - \frac{\bar{M}^2}{M_B^2}) + 1] \bar{M}^2 e^{-\bar{M}^2/M_B^2}} \quad (21)$$

$$A_P = -\frac{\frac{A}{4^3\pi^2 L^{4/9}}(\frac{76}{3}M_B^6 amE_2 - \sqrt{2}E_4 M_B^{10} F_\pi) + \tilde{\lambda}_N \tilde{\lambda}_\Sigma c_b^- \bar{M}^2 e^{-M^2/M_B^2}}{\tilde{\lambda}_N \tilde{\lambda}_\Sigma \bar{M}^2 e^{-\bar{M}^2/M_B^2}} \quad (22)$$

The parameters that used in this work are A = 3.57 10^{-6} GeV^{-2}, m=.15 GeV, a=.55 GeV^3, and L = 0.621 ln(10 M_B). The experimental values are $A_S = (4.27 \pm 0.02) \times 10^{-7}$, $A_P = (-1.52 \pm 0.16) \times 10^{-7}$, $A_S/A_P \approx -2.8$. The single-pole parameters which give stable solutions for A_s are

c_a^- = (-6.37+10.7 M_B)$\times 10^{-7}$, and for A_p are c_b^- = (1.37 - .25 M_B)$\times 10^{-7}$. The solutions are shown in Fig. 4.

2.3 Σ_+^+ Decay in an External Field

For Σ^+ we take

$$\eta_\Sigma^+ = \epsilon^{abc}[u^{aT}C\gamma_\mu u^b]\gamma^5\gamma^\mu s^c \ . \tag{23}$$

The relevant diagrams are shown In Fig. 5

$$\begin{aligned}
Fig.5a &: \ 0 \\
Fig.5b &: \ -\frac{2A<\bar{q}q>p^2\ln p^2}{(4\pi)^4}(\frac{4}{3}\not{p}+\frac{25}{6}m)(1-\gamma_5) \\
Fig.5c &: \ \frac{2A<\bar{q}q>p^2\ln p^2}{(4\pi)^4}(\frac{4}{3}\not{p}+\frac{3}{2}m)(1-\gamma_5) \\
Fig.5d &: \ 0 \\
Fig.5e &: \ 0 \\
Fig.5f &: \ \frac{8Ap^4\ln p^2}{(4\pi)^6\epsilon}[p^2(1-\frac{3}{2}\gamma\epsilon+\frac{25}{6}\epsilon-\frac{9}{8}\epsilon\ln p^2)-4m\not{p}(1-\frac{3}{2}\gamma\epsilon+\frac{73}{16}\epsilon-\frac{9}{8}\epsilon\ln p^2)](1+\gamma_5) \\
Fig.5g &: \ -\frac{8A<\bar{q}q>p^2\ln p^2}{(4\pi)^4\epsilon}[2\not{p}(1-\gamma\epsilon+\frac{3}{2}\epsilon-\frac{\epsilon}{2}\ln p^2)+m(1-\gamma\epsilon+\frac{\epsilon}{2}-\frac{\epsilon}{2}\ln p^2)](1-\gamma_5) \\
Fig.5h &: \ -\frac{4A<\bar{q}q>p^2\ln p^2}{(4\pi)^4\epsilon}[\not{p}(1-\gamma\epsilon+\frac{5}{4}\epsilon-\frac{\epsilon}{2}\ln p^2)+2m(1-\gamma\epsilon+\frac{3}{2}\epsilon-\frac{\epsilon}{2}\ln p^2)](1-\gamma_5) \\
Fig.5i &: \ 0 \\
Fig.5j &: \ \frac{4A<\bar{s}s>p^2\ln p^2}{(4\pi)^4\epsilon}[\not{p}(1-\gamma\epsilon+\frac{9}{4}\epsilon-\frac{\epsilon}{2}\ln p^2)](1+\gamma_5)
\end{aligned} \tag{24}$$

Once again we need to carry out vertex renormalizations for Figs. 5f, g, h and j. We omit the details and simply show the results. As for the Σ^- decays we omit the $\not{p}$ and $\not{p}\gamma_5$ terms. We find

$$\Pi = -\frac{10}{3}\frac{Am<\bar{q}q>p^2\ln p^2}{(4\pi)^4}(1-\gamma_5)+8(\frac{8}{3}-\frac{\gamma}{2})\frac{Ap^6\ln p^2}{(4\pi)^6}(1+\gamma_5) \ . \tag{25}$$

After taking a Borel transform, adding anomalous dimensions and continuum contributions, this becomes

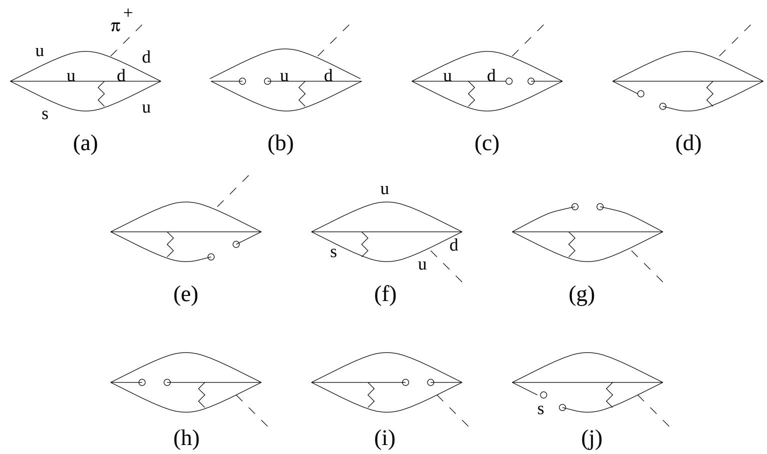

Fig. 5 Processes for Σ^+ decay into a neutron with an external π^+ field

$$\Pi = \frac{10}{3}\frac{Am < \bar{q}q >}{(4\pi)^4}(1-\gamma_5)M_B^4 E_2 L^{-4/9} - 48(\frac{8}{3}-\frac{\gamma}{2})\frac{A}{(4\pi)^6}(1+\gamma_5)M_B^8 E_4 L^{-4/9} \ . \tag{26}$$

$$A_S = -\frac{\frac{A}{4^3\pi^2 L^{4/9}}(\frac{20}{3}M_B^6 amE_2 + 128(1-\frac{3\gamma}{16})E_4 M_B^{10}) - \tilde{\lambda}_N\tilde{\lambda}_\Sigma c_a^+ \bar{M}^2 e^{-M^2/M_B^2}}{\tilde{\lambda}_N\tilde{\lambda}_\Sigma[2(1-\frac{\bar{M}^2}{M_B^2})+1]\bar{M}^2 e^{-\bar{M}^2/M_B^2}} \tag{27}$$

$$A_P = -\frac{\frac{A}{4^3\pi^2 L^{4/9}}(\frac{20}{3}M_B^6 amE_2 - 128(1-\frac{3\gamma}{16}E_4 M_B^{10}) + \tilde{\lambda}_N\tilde{\lambda}_\Sigma c_b^+ \bar{M}^2 e^{-M^2/M_B^2}}{\tilde{\lambda}_N\tilde{\lambda}_\Sigma \bar{M}^2 e^{-\bar{M}^2/M_B^2}} \tag{28}$$

Experimentally, $A_S = (0.13\pm0.02)\times10^{-7}, A_P = (44.4\pm0.16)\times10^{-7}, A_S/A_P \approx 0.003$. The single-pole parameters which give stable solutions for A_s are c_a^+ = (-1.42 + 9.8 M_B)$\times10^{-7}$, and for A_p are c_b^+ = (-46.3 +9.5 M_B)$\times10^{-7}$.The solutions are shown in Fig.6.

2.4 Σ^-_- in the Pion Matrix Method

The pion matrix method [9, 10, 11] is simpler than the external field one and does not require any renormalizations. Instead of treating quarks propagating in an external field with a correlator defined between the vacuum states,

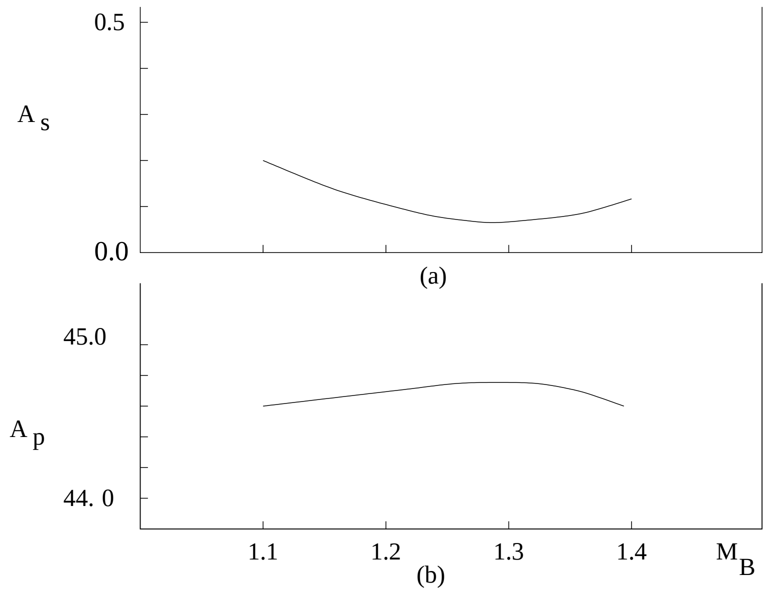

Fig. 6 Σ^+_+ decay (a) A_s , (b) A_p

the correlator is defined with a one-pion final state. This is also the starting point for light-cone sum rules which have been used for the pion form factor[12] and the pion wave function[13]. One makes use of the correlator

$$\Pi = i \int d^4x \, e^{iq\cdot x} < \pi(p=0) \mid T[\eta_a(x) H_W \bar{\eta}_b(0)] \mid 0 > , \qquad (29)$$

rather than Eq. (9) for the external field. For the Σ^- the corresponding non-vanishing diagrams are shown in Fig. 7. Carrying out the required algebra and integrations, and doing a Borel transform, we obtain

$$\begin{aligned} Fig7a &: \frac{4}{3} A \frac{<\bar{q}q>}{(4\pi)^2} E_1 M_B^2 < \pi^- \mid \bar{d} i\gamma_5 u \mid 0 > (\not{p} + 2m)(1-\gamma_5) , \\ Fig.7b &: -\frac{4}{3} A \frac{<\bar{q}q>}{(4\pi)^2} E_1 M_B^2 < \pi^- \mid \bar{d} i\gamma_5 u \mid 0 > (\not{p} + 2m)(1-\gamma_5) , \\ Fig.7c &: -\frac{\sqrt{2}}{3} A F_\pi \frac{1}{(4\pi)^4} < \pi^- \mid \bar{d} i\gamma_5 u \mid 0 > (4m M_B^4 E_2 L^{-4/9} \not{p} - 2M_B^6 E_3 L^{-4/9})(1+\gamma_5) . \end{aligned} \qquad (30)$$

Note that the contributions of Figs 7a and 7b cancel exactly, so that only that from Fig 7c remains and it is seen that, in contrast to the external field

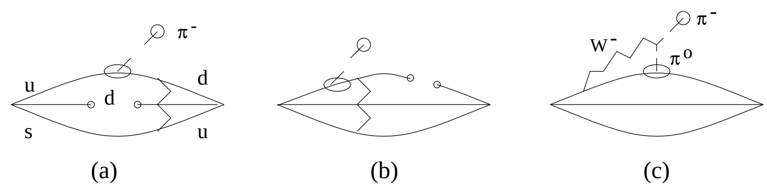

Fig. 7 Processes for Σ^- decay into a neutron and a π^- by matrix method

method, both S and P wave amplitudes have the same sign. The phenomenologic side is the same as before so that we obtain

$$A_S \;=\; \frac{-\frac{A}{16\pi^2}\frac{a}{f_\pi}\frac{M_B^8 E_3}{L^{4/9}} + \tilde{\lambda}_N \tilde{\lambda}_\Sigma c_a^- \bar{M}^2 e^{-M^2/M_B^2}}{\tilde{\lambda}_N \tilde{\lambda}_\Sigma [2(1-\frac{\bar{M}^2}{M_B^2})+1]\bar{M} e^{-\bar{M}^2/M_B^2}}\;, \tag{31}$$

$$A_P \;=\; +\frac{\frac{A}{16\pi^2}\frac{a}{f_\pi}\frac{M_B^8 E_3}{L^{4/9}} - \tilde{\lambda}_N \tilde{\lambda}_\Sigma c_b^- \bar{M}^2 e^{-M^2/M_B^2}}{\tilde{\lambda}_N \tilde{\lambda}_\Sigma \bar{M}^2 e^{-\bar{M}^2/M_B^2}}\;, \tag{32}$$

where we have put $F_\pi = 1$ and used the soft pion matrix element

$$< 0 \mid \bar{u} i \gamma_5 d \mid \pi^- >= \sqrt{2}\frac{< \bar{q}q >}{f_\pi} = -\frac{\sqrt{2}a}{(2\pi)^2 f_\pi}, \tag{33}$$

with f_π = .093 GeV in Eqs.(31,32).

The single-pole parameters which give stable solutions for A_s are c_a^- = (-4.3 + 12.0 M_B) $\times 10^{-7}$, and for A_p are c_b^- = (1.13 +3.67 M_B)$\times 10^{-7}$. The results are similar to those shown in Fig.4.

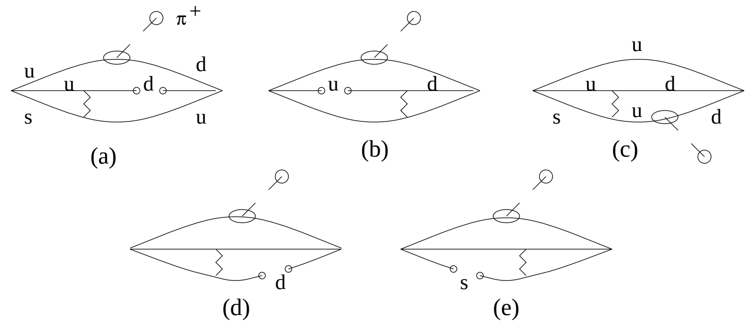

Fig. 8 Processes for Σ^+ decay into a neutron and a π^+ by matrix method

2.5 Σ^+_+ Decay in the Pion Matrix Method

The contributing diagrams are shown in Fig. 8

$$
\begin{aligned}
Fig8a &: \frac{4}{3}A\frac{<\bar{q}q>}{(4\pi)^2}E_1M_B^2 <\pi^+\mid \bar{u}i\gamma_5 d\mid 0> (\not p + 2m)(1-\gamma_5)\ , \\
Fig.8b &: -\frac{2}{3}A\frac{<\bar{q}q>}{(4\pi)^2}E_1M_B^2 <\pi^+\mid \bar{u}i\gamma_5 d\mid 0> (2\not p + m)(1-\gamma_5)\ , \\
Fig.8c &: -4\frac{A}{(4\pi)^4} <\pi^+\mid \bar{u}i\gamma_5 d\mid 0> (\frac{4}{3}M_B^6E_3 + mM_B^4E_2\not p\{(\frac{1}{\epsilon}-\gamma+\frac{31}{12} \\
&\quad +[\ln M_B^2 + (1-\gamma)])\}(1+\gamma_5)\ . \\
Fig.8d &: -\frac{2}{3}A\frac{<\bar{q}q>}{(4\pi)^2}E_1M_B^2 <\pi^+\mid \bar{u}i\gamma_5 d\mid 0> (2\not p + m)(1-\gamma_5)\ , \qquad (34) \\
Fig.8e &: \frac{4}{3}A\frac{<\bar{s}s>}{(4\pi)^2}E_1M_B^2 <\pi^+\mid \bar{u}i\gamma_5 d\mid 0> (\not p + 2m)(1-\gamma_5)\ ,
\end{aligned}
$$

As usual, we discard the odd sum rules; in that case no renormalization is required. For the even sum rule we obtain

$$
\Pi = \frac{4}{3}\ \frac{Am}{(4\pi)^2} <\pi^+\mid \bar{u}i\gamma_5 d\mid 0> M_B^2E_1L^{-4/9}(<\bar{q}q> + 2<\bar{s}s>)(1-\gamma_5) \qquad (35)
$$

$$
-\frac{16}{3}\ \frac{A}{(4\pi)^4} <0\mid \bar{d}i\gamma_5 u\mid \pi^+> M_B^6E_3L^{-4/9}(1+\gamma_5) \qquad (36)
$$

For $<\bar{s}s>$ we take $0.8<\bar{q}q>$. We thus obtain for A_S and A_P

$$A_S \;=\; \frac{\frac{\sqrt{2}AM_B^4 a}{4\pi^2 f_\pi L^{4/9}}[\frac{2.6}{3}maE_1 + M_B^4 E_3] + \tilde{\lambda}_N\tilde{\lambda}_\Sigma c_a^+ \bar{M}^2 e^{-M^2/M_B^2}}{\tilde{\lambda_N}\tilde{\lambda_\Sigma}[2(1-\frac{\bar{M}^2}{M_B^2})+1]\bar{M}^2 e^{-\bar{M}^2/M_B^2}} \qquad (37)$$

$$A_P \;=\; \frac{\frac{\sqrt{2}AM_B^4 a}{4\pi^2 f_\pi L^{4/9}}[\frac{2.6}{3}maE_1 - M_B^4 E_3]}{\tilde{\lambda_N}\tilde{\lambda_\Sigma}\bar{M}^2 e^{-\bar{M}^2/M_B^2}} - c_b^+. \qquad (38)$$

The single-pole parameters which give stable solutions for A_s are $c_a^+ =$ (1.0-22.3 M_B)$\times 10^{-7}$, and for A_p are $c_b^+ =$ (-39.8 -19.7 M_B)$\times 10^{-7}$. The results are similar to those shown in Fig.6.

3 Discussion

In a recent work by Borasoy and Holstein[3] it was found that by including resonances in the chiral perturbation theory approach to nonleptonic hyperon decays one can obtain fits to data, which does not seem possible if they are not explicitly included. In the QCD sum rule method we find it is also necessary to include resonances or single pole terms and have done so.

In summary, we have used QCD sum rules and two 2-point formalisms to examine the Σ^+_+ and Σ^-_- nonleptonic decays. We have assumed that pion kinetic energy effects are small and have concentrated on the sum rules for which we expect these effects to be of order $\triangle M/\bar{M} \ll 1$.We have omitted gluonic corrections, in part because we show that the single pole (resonance) contributions can be adjusted to fit the data, and the changes in these parameters when gluonic corrections are included would not be large. Stable and consistent fits to the decay amplitudes are obtained with resonance contributions of similar magnitude as those used in chiral perturbation theory fits. These contributions are not known and are large, so that at the present stage the method has no predictive power, which is also true of the chiral perturbation theory calculations. Our results, however, parameterize quite specific terms in the dispersion relations for the correlation functions that we have used, and might be useful for predicting related reactions.

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